THE EFFECT OF 5E LEARNING CYCLE MODEL IN TEACHING TRIGONOMETRY ON STUDENTS’ ACADEMIC ACHIEVEMENT AND THE PERMANENCE OF THEIR KNOWLEDGE

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ABSTRACT

In this study, the effect of 5E learning cycle model, based on the constructivist approach, which is used for teaching trigonometry in 10th grade of elementary mathematics education, on the students’ academic achievement and on the permanence of their trigonometry knowledge is investigated. The participants of this research are 10th grade students registered for spring semester of 2010-2011 academic year to an Anatolian high school in Kastamonu. These students divided into two equal groups, a control and an experimental group. The students in the experimental group took the course about trigonometry from the researcher in an environment where the 5E learning model based on the constructivist approach is used. The students in the control group took the same course from their mathematics teacher in an environment where the activities of official mathematics curriculum are used. The statistical findings of the research show that the experimental group students’ scores of academic achievement and permanence of trigonometric knowledge are higher than those in the control group. The difference between these groups is statistically significant and is in favor of the experimental group.

Key Words: Mathematics Education, Trigonometry Teaching, 5E Learning Cycle Model.

INTRODUCTION

Teaching is “the process of using appropriate method, staff and material in order to reach in the most effective manner to the predetermined goals”. Teaching is a conscious and purposeful activity. Oriented to the predetermined goals and aimed to earn desirable behavior, teaching activities usually take place in the institutes of educations. Planned, controlled and organized teaching activities that occur in schools are called instruction (Uzun, 2002).

According to Renate and Geoffrey Craine (1990), there are some basic points about effective ways which can be learned in brain-based researches in learning and teaching:

- Previous experiences and significations effect the brain in the way that it operates new experiences and organizes new information. Our senses and our knowledge share an important relation. We can think only one thing at a moment; in contrast, our brain can realize many brain functions and organize many stimuli at the same time.

- According to Selçuk(2000), the new concepts, information and principles to be learned can only make sense when they are related to the previous ones. Since the quality of learning, as a process in education, depends on the complete, in every way, participation of the individual to this process, in actual learning approach, the
student-centered instruction is accepted as the basic principle. Designed according to students’ interests, needs, talents and skills, the student-centered teaching environment creates successful individuals (Altunay, 2004).

Piaget claims that a child can learn a lot of things when he/she plays with mud, and in the Piaget’s view, when a child interacts actively with his/her environment, he/she gives meaning better to the events and objects (Onur, 1993). If an individual associates meaningfully pieces of knowledge in the active interaction and forms connections, he can classify, organize and code this knowledge easily. The knowledge gathered in this way can be kept in memory for long time (Bruner, 1972).

5E Learning Cycle Model

Grounded on learning cycle, 5E model was developed by Robert Bybee. According to Bybee (1997), the foundation of this model was affected by works of German philosopher Freidrich Herbart. Furthermore, in his view, this model is based on the ground of John Dewey and Jean Piaget. As a very frequently used model in constructivist learning approach, 5E learning cycle model’s name comes from the number of its phases and the initials of each phase. These five phases are:

- Engage/Enter
- Explore
- Explain
- Elaborate
- Evaluate

The 5E learning cycle model is a constructivist model which provides learning a new concept or comprehension deeply a known concept. This model which increases students’ merak of research, by satisfying expectations of students, consists of active research’s skills and activities that are necessary for knowledge and comprehension (Ergin, Ünsal and Tan, 2006). The 5E model targets at the discovery and the association with previous knowledge of new concepts by students. With the aid of planned and applied learning-teaching activities, students form themselves their own knowledge about a specific problem (http://www.bap.gazi.edu.tr/projeler/gefp/devrim.htm).

5E model motivates students to be included into a topic by several phases of learning, to explore a subject, to be given a definition for their experiences, to obtain more detailed information about their learning and to evaluate it (Wilder and Shuttleworth, 2005). 5E learning cycle is one of the complete constructivist models in the cases of research-based learning or brain-storming which are used in the classroom (Campbell, 2000). Students have to think creatively and complex for overcoming problems and difficulties and as a result, they have to think in an integrative manner in order to unify their thoughts. This situation can only occur when students have higher order thinking skills. These skills are called also critical thinking skills.

Grounded on the constructivist approach, 5E learning and teaching model includes higher order thinking skills. Stimulating students to explore, to inquiry, to get experience, 5E model transmits also the critical thinking skill to students (Ergin, 2006). 5E model is a learning cycle model that facilitates learning and creates beneficial opportunities for students while learning (Lorsbach, 2006).

Phases of 5E Learning Cycle Model

1. Engage/Enter

The purpose of this phase is to focus students’ attention on the topic. Asking pointed questions, explaining a scenario, a demonstration of an event, showing a picture or making a discussion can be used to focus the students’ attention on the tasks that will follow and connections to past learning and experience can be invoked (http://www.miamisci.org/ph/). In this phase, there is no lecturing and the subject of course is not told
to students. Students should be encouraged and the tasks for learning are defined. In this phase, past experiences are connecting with actual experiences. The basis of work for upcoming activities is organized.

Students derive some questions and try to find answers to them. For teachers, this phase provides opportunities for determining their students’ misconceptions (Balcı, 2005). In addition, this phase can be used to create disequilibrium in students’ mind and to motivate students for using related real-life situations. In this phase, where teachers ask questions for arousing students’ interest about topic and for motivating them, teachers avoid defining and making explanations about concepts (Carin and Bass, 2001). This aroused interest leads students to the “explore” phase where they use concrete experiences for observation, collect information, test and reformulate hypotheses (Wilder and Shuttleworth, 2005).

2. Explore
Motivated to the subject in engage/enter phase, student makes some research activities which consist of gathering data, observation, guessing and testing them and making hypotheses (Wilder and Shuttleworth, 2005). The step, where students try to understand and to explore the subject via only their own experiences and thoughts, by making and testing hypotheses, can last short or long (Temizyürek, 2003). After giving short explanations about the activity that will follow, teacher can give to student a concept map to fill out, may want students to make experiments or may make organize a demonstration. Students can work in small groups for this activity (Lord, 1999).

The phase that students make the most activities is the explore phase. In this phase, students try to solve the given problem by working, discussing and experimenting in groups. Meanwhile, teachers should only guide students, not participate entirely to the students’ work. While guiding, if a teacher sees students’ mistake, he/she should not directly correct it, but should give some hints or show some ways to students for correcting themselves. As students interact with each other, they are not passive in this process. They can announce freely their opinions, test every idea and enregister the results. They try to interpret and explain the results of their observations (Carin and Bass, 2001).

3. Explain
In the explain phase, students explain scientifically the results obtained from their observations and data. Appropriate verbal repertoire should be associated with students’ data and experiences (Wilder and Shuttleworth, 2005). A representative in each group, formed in the explore phase, explain the results of their work and let their friends discuss about them. The explain phase is a teacher-centered phase in 5E model; because teachers become active for correcting mistakes and completing the missing parts in students’ results. Teachers may choose lecture method or may use another interesting method like showing a film or a video, making a demonstration or giving an activity which leads students to define their work or to explain their results. In this phase, teachers give formal definitions and scientific explanations. Furthermore, by giving explanations in basic knowledge level to students, teachers, whenever possible, help them to unify together their experiences, to explain their results and to form new concepts (Bybee, 1997). The aim of this phase is to correct mistakes in students’ findings before the next phase (Hançer, 2005).

4. Elaborate
In this phase, students can practice their new knowledge, suggest solutions, create new problems and make decisions and/or introduce logical implications. These situations can be realized by presenting a new research activity or by extending the activities done in the explore phase (Wilder and Shuttleworth, 2005). Working in groups also in this phase, students are close to end up the asked problem. The groups present and explain their final situations. This phase can be considered as the extension of research step because of the existence of supplement problems. Small group works or whole class discussions provide opportunities for students to understand the subject, to defend and to present their thoughts. To use the new learned concept in different situations or to repeat several times the applications related to the concept is necessary for being
put in the long term memory and being permanent. The elaborate phase is important because the new learned is corroborated and its permanence is supported.

5. Evaluate
The evaluate phase has the importance in determining whether or the students learn the concept correctly in scientific context and reflect it to the context. This phase may be realized in formal or informal method (Wilder and Shuttleworth, 2005). In this phase, some evaluations are made for revealing students’ constructed knowledge. Students may answer to oral questions, make short summaries, fill out empty maps, read graph and evaluate tables. Furthermore, students are asked to associate what they have learned, with real life situations. This phase is the phase where students may exhibit their attitudes about learning and may change their thinking style or behaviors. The evaluation is realized over and over whenever teachers and students try to control the development in reaching to new comprehension (Hançer, 2005). This phase reveals how students constructed scientific knowledge and they generalize it to other situations (Wilder and Shuttleworth, 2005).

The phases of 5E learning cycle model are schematized as below:

![Phases Of 5E Learning Cycle Model](image)

**Research Problem**
In teaching trigonometry, does 5e learning cycle model have an effect on students’ academic achievement and the permanence of their knowledge?

**Purpose of Research**
The common purpose of researches in education is to focus on how to increase students’ achievement. Participatory learning grounded on students’ participation to course has to be realized in order to obtain the permanence of the learning. Constructive approach has a great importance in the formation of meaningful and complete learning. With the aid of constructivist approach, students can easily stay away from the rote learning and make connections between subjects by exploring.

The purpose of this study is to investigate the effect of, 5e learning cycle model in teaching trigonometry on students’ academic achievement and the permanence of their knowledge.

**Significance of Research**
Trigonometry is one of the important components of mathematics. Students have usually difficulties in this subject. In primary, elementary, high school and higher education, students that meet with trigonometric
concepts like sinus, cosines, tangent and cotangent, have difficulties to associate them with real life situations and don’t know where these concepts come from. If the formation, the association with real life situations and the importance of using of these concepts are explained to students, these concepts may be learned better. In more positive words, students can consider trigonometry, not as an external concept to mathematics, but as one of its components (Adamek, Penkalski and Valentine, 2005).

Nowadays, trigonometry is used in physics for understanding the space, in engineering and in chemistry. In mathematics, it is used first of all in calculus and in other branches such as linear algebra and statistics. The importance of this research relies on the fact that it is an evaluation of effectiveness of an alternative learning model in teaching correctly trigonometry. Furthermore, by showing how 5E learning cycle model can be applied in a learning environment, this research is also important.

Restrictions
This research is limited by the basic concepts of trigonometry in 10th grade high school mathematics. During the application of research, the equality of course hours of both control and experimental group is satisfied; but no specific precaution has taken about scheduling. The research is limited by 8 weeks (4*8=32 course hours).

METHODOLOGY
In this section, the information about the design of the research, the working group, the data gathering and the data analysis are mentioned.

Design of the research
The design of this experimental research is chosen as randomized pretest-posttest control group design. In this design, there exist two groups formed by random assignment. One of them is used as an experimental, the other as a control group. The students in the experimental group took the course about trigonometry from the researcher in an environment where the 5E learning model based on the constructivist approach is used. The students in the control group took the same course without intervention of the researcher, from their mathematics teacher in an environment where the activities of official mathematics curriculum are used.

Working Group
The working group of this research consists of 49 students in 10th grade registered for spring semester of 2009-2010 academic year to MK Anatolian high school in Kastamonu. In the experimental group, there are 25 students (13 girls, 12 boys) and in the control group, there are 24 students (12 girls and 12 boys). When the experimental and control groups are determined, their mathematics’ grades for autumn semester and their scores in the pre-test about dependent variable (academic achievement in trigonometry) are taken into consideration and randomly the class 10-D and 10-C are selected respectively as experimental and control group. If the effect of a new teaching method on learning is investigated in a research, the groups’ equalization in gender and in past achievement is crucial (Büyüköztürk et al., 2008).

Students’ mathematics’ grades for autumn semester of 2009-2010 academic year
Independent samples t-test is used to determine whether there is a statistically significant difference between experimental and control group students’ mathematics’ grades for autumn semester of 2009-2010 academic year.
Table 1: Independent Samples T-Test For Experimental And Control Group Students’ Mathematics’ Grades For Autumn Semester

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of students (N)</th>
<th>Mean (\bar{X})</th>
<th>Sd.</th>
<th>df.</th>
<th>t</th>
<th>Sig. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>25</td>
<td>3,00</td>
<td>1,63</td>
<td>47</td>
<td>0,773</td>
<td>0,444</td>
</tr>
<tr>
<td>Control</td>
<td>24</td>
<td>3,31</td>
<td>1,52</td>
<td></td>
<td></td>
<td>p&gt;0,05</td>
</tr>
</tbody>
</table>

As seen in table 1., there is no statistically significant difference between experimental and control group students’ mathematics’ grades \([t(47) = 0,773, p>0,05]\). As a result, before the start of experimental application, these two groups are considered as equal in their mathematical pre-knowledge.

Pre-test scores of students in the experimental and control group

Independent samples t-test is used to determine whether there is a statistically significant difference between experimental and control group students’ scores in the achievement test for trigonometry used as pre-test.

Table 2: Independent Samples T-Test For Experimental And Control Group Students’ Scores In The Pre-Test

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of students (N)</th>
<th>Mean (\bar{X})</th>
<th>Sd.</th>
<th>df.</th>
<th>t</th>
<th>Sig. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>25</td>
<td>4,640</td>
<td>1,350</td>
<td>47</td>
<td>0,183</td>
<td>0,856</td>
</tr>
<tr>
<td>Control</td>
<td>24</td>
<td>4,708</td>
<td>1,267</td>
<td></td>
<td></td>
<td>p&gt;0,05</td>
</tr>
</tbody>
</table>

As seen in table 2., there is no statistically significant difference between experimental and control group students’ scores in the achievement test for trigonometry \([t(47) = 0,183, p>0,05]\). As a result, before the start of experimental application, these two groups are also considered as equal in academic achievement for trigonometry.

Data Gathering

Academic achievement test is used to gather necessary data for the statistical analysis of research problem and to determine the effect of using 5E learning cycle model in teaching trigonometry on students’ academic achievement. Consisting of 50 questions, a test is prepared for measuring each acquisition of trigonometry by at least 2 questions and it is applied to 120 students from upper grade. The content validity of the test are provided by a commission of five persons which are specialists in the field, in program development and in evaluation. After the pilot application, an item analysis is done, the item distinctiveness and the item difficulty index are checked. By eliminating depending on their proprieties, the final version of test which consists of 28 questions is prepared. The KR 20 reliability constant of the test is found as 0,88.

This academic achievement test is used for several purposes: first of all, as a pre-test, it is applied to students in order to determine their pre-knowledge and readiness about trigonometry. Moreover, as a post-test, it is used in order to determine the effect of teaching method. Finally, as a permanence test after one month later than post-test, it is applied in order to investigate the permanency of knowledge.
Procedure of experimental application

Registered for spring semester of 2009-2010 academic year to an Anatolian high school in Kastamonu, 10th grade students mathematics’ grades for autumn semester and their scores in the pre-test about dependent variable which is the academic achievement in trigonometry are took into consideration. As a result, the groups are found equal in mentioned variables and randomly the class 10-D and 10-C are selected respectively as experimental and control group.

The students in the experimental group took the course about trigonometry from the researcher in an environment where the 5E learning model based on the constructivist approach is used. Before the beginning of the courses, course plans are prepared, checked and corrected by the specialists in that field.

A pilot study for testing the applicability of the course plans is done with another group that is independent of research. The research lasted 8 weeks (8*4=32 course hours).

The classroom and computer laboratories are used as environments for courses. Some teaching materials, which the mathematics teacher considered as necessary, are used by him. At the end of experimental application, the post-test is applied to both of the groups.

One month later, the academic achievement test is applied one more time as a permanence test.

Analysis of data

For the analysis of data obtained in research, SPSS 15.0 (Statistical Package for the Social Sciences) package program is used. All the analysis is made in computer and statistical analysis methods, which are appropriate to the properties of data, are used.

FINDINGS

In this section, the findings from the analysis by SPSS 15.0 of data obtained in research are presented with tables and graphics. Comparison of experimental and control group students’ post-test scores The mean and the standard deviation of post-test scores of students in the experimental group, where the 5E learning model based on the constructivist approach is used, and of those in the control group, where the traditional teaching activities (lecture method, question-answer method etc.) are used, in academic achievement test about trigonometry are given in the table 3.

Table 3: Independent Samples T-Test For Experimental And Control Group Students’ Scores In The Post-Test

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of students (N)</th>
<th>Mean ((\bar{x}))</th>
<th>Sd.</th>
<th>df.</th>
<th>t</th>
<th>Sig. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>25</td>
<td>20,76</td>
<td>3,205</td>
<td>47</td>
<td>5,677</td>
<td>0,000</td>
</tr>
<tr>
<td>Control</td>
<td>24</td>
<td>16,00</td>
<td>2,620</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in table 3., the mean of post-test scores of students in the experimental group is found as \(\bar{x} = 20,76\) and the mean of those in the control group is found as \(\bar{x} = 16,00\). It is determined that he mean of academic achievement of the experimental group is higher than the one of control group. Independent samples t-test is used to determine whether this difference is statistically significant. As a result, as t score is determined as
5,677 with df = 47 and p=0.000, the difference between means is found statistically significant at the level of 0.05 significance.

These findings show that there is a learning level difference in favor of the experimental group. Thus, the 5E learning model based on the constructivist approach used in the experimental group is more effective in teaching trigonometry that the traditional teaching methods used in the control group.

**Findings about the permanence test scores**

In this section, the findings about the students’ scores in the permanence test which is used to determine the permanence of students’ knowledge about trigonometry are given. The permanence test is applied 4 weeks later than the post-test. The experimental and control group students’ score in the permanence test are shown in the table 4.

**Table 4: Independent Samples T-Test For Experimental And Control Group Students’ Scores In The Permanence Test.**

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of students (N)</th>
<th>Mean (X)</th>
<th>Sd.</th>
<th>df.</th>
<th>t</th>
<th>Sig. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>25</td>
<td>18,840</td>
<td>2,718</td>
<td>47</td>
<td>6,446</td>
<td>0.000</td>
</tr>
<tr>
<td>Control</td>
<td>24</td>
<td>13,291</td>
<td>3,290</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As seen in table 4., there is a statistically significant difference between experimental and control group students' scores in the permanence test in favor of the experimental group [t (47) =6,446 and p=0,000 < 0,05]. The permanence test scores of students in the experimental group, where the 5E learning model based on the constructivist approach is used, is higher than those in the control group. As a result, it can be interpreted that the trigonometry learning by the activities appropriate to the 5E learning model based on the constructivist approach is more permanent that the traditional teaching.

**DISCUSSION AND RESULTS**

As a result of this research, it is found that 5E learning cycle model effects not only the students’ achievement but also the permanence of knowledge. The researches about 5E learning cycle models in both domestic and international literature are generally made in science education. Saka(2006), Lawson(2001), Balçi (2005), Bleicher (2001), Akar(2005) and Özsevgeç (2007) have researched the effect of 5E learning cycle model on academic achievement. The findings of this research show similarity to these mentioned researches.

A statistically significant difference is determined between experimental and control group students’ scores in the post-test. By the analysis of findings, when their post-test scores are took into consideration, students in the experimental group, where the 5E learning model based on the constructivist approach is used, are found as more successful than those in the control group. A statistically significant difference is determined between experimental and control group students’ scores in the permanence test. It is determined that this difference is in favor of the experimental group.

In accordance with these results, some suggestions, thought as beneficial, are given below:

Mathematics teachers should provide opportunities to students for learning by exploring and reaching themselves to knowledge. The students should be asked to give all reasons of solving steps of the given problem. Whenever possible, the passage to the application should be done directly by students and also during the application, the mistakes and errors should be found directly by them. Teachers should only orientate students, help them.
Teachers should be careful about choosing teaching activities and working papers, which they want to use in the application of 5E learning model, in the sense that these materials should be attractive and appropriate to the students’ level and also they provide opportunities for students to construct their own knowledge. This research is limited by the trigonometry subject. The effect of 5E learning model on other subjects in mathematics can be researched.

The students of the last grade in faculty of education should be asked to analyze and/or to research such models (5E learning model) as homework or project etc. Moreover, they should be asked to prepare course plans according to these models and to apply them, whenever possible. These course plan examples should be evaluated by the authorities. Such works provide opportunities for pre-service teachers to learn about these methods in other subjects of mathematics and also these teachers may use these methods when they begin to work.

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REFERENCES


APPENDIX

EXAMPLE OF COURSE PLAN IN TRIGONOMETRY TEACHING BASED SE LEARNING CYCLE MODEL

Acquisitions : Students state the cosine law and make applications related to this law
Pre-acquisitions :
Students state sine law and find the formulate the area of a triangle
Students state the trigonometrical ratio of acute angles in a right triangle
Duration : 45+45=90 minutes

ENGAGE

Before beginning of learning new things, a human being has to be aware of what he already knows. Therefore, the first thing that teacher have to do is to help students to determine what they know about subject. Students should be led to think on given problems and they should be encouraged to make brainstorming by saying their ideas, to share their thoughts and to make connections with the new subject. An environment where students aren’t judged and can declare freely their own ideas has to be created. In this phase, students are motivated and teacher should leave question marks over their minds. Moreover, teacher may stimulate students to interrogate their ideas by these question marks. Meanwhile, teacher can observe the readiness of students and can increase students’ motivation for learning. Therefore, by stimulating students’ interest, teacher prepares them for the next phase. Teacher shows the following picture to students on the screen and asks the prepared questions. The aims of these questions are to determine whether students know the sine law, as one of the pre-acquisitions, and the trigonometric functions, to determine misconceptions about them and to create the infrastructure of why to use the cosine law.

According to the data in the figure above,
How can you find the distance between two houses?
Can you obtain the solution by drawing the height of the side opposite to the corner where the angle is 110°?
Can you find the unknown side (the distance between two houses) by using sine law?
Can you solve the problem by using the area formula in the sine law where two sides and the angle between them are given?

EXPLORE

In this phase, teacher provides an environment where students discuss between them and try to find answers to the questions in their minds formed in the previous phase. Teacher strolls among the students, observe them and asks if needed some challenging questions to students. Students are provided some opportunities to construct themselves the knowledge in question. In this phase, by using purposive materials, students try to reach the target knowledge. Students are asked to work in pairs and are given enough time to complete activities. Students create ideas and take notes on working papers. The activity paper below is distributed to each group composed of 3 students and they are asked to make necessary operations.
In the figure above, supposing that $AB = c$, $BC = a$ and $\angle ABC = \beta$ are given, try to find $AC = b$.

The height $h$ of the side is $AB$ already drawn.

If $BD$ equals to $x$, how can you write $AD$ in terms of $x$?

Using the Pythagorean relation, write $a^2$ and $b^2$ in two right triangles.

\[ a^2 = \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \}
In this phase, teachers give formal definitions and scientific explanations. Furthermore, by giving explanations in basic knowledge level to students, teachers, whenever possible, help them to unify together their experiences, to explain their results and to form new concepts.

In this phase, teacher and students listen to the explanations of each group. Then, teacher corrects the possible wrong concepts and explains what the cosine law is and why it is necessary. Students take notes on their notebook.

Given any triangle $\Delta ABC$;

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

These equalities hold and they are called as Cosine Law.

**ELABORATE**

In this phase, students apply the knowledge or the problem solving approach reached together to new events or problems. Through this way, they learn new concepts that haven’t been in their minds.

Students are asked to be more correct and more responsible for applying the learned knowledge to the new situations. Students are stimulated to use formal terms and definitions and also to exhibit their comprehension. By asking different types of questions, teacher makes students use what they learned, in new situations. By interchanging their ideas in the group, students try to answer the questions.

Students, now, can solve the questions in the engage phase. Furthermore, teacher help students to make connections between the cosine law and the older knowledge, by asking questions related to them.

For this, different typed questions like below can be asked to students:

In a right triangle, by using the cosine law, show that the Pythagorean relation is one of special situation of the cosine law.

By using the cosine law, how can you prove that a triangle with sides $a = 2cm$, $b = 3cm$ and $c = 11cm$ can’t be drawn?

According to the above figure, calculate $|AB|$ by using the cosine law.
EVALUATE

Up to this phase, students’ activities are evaluated through the process. But in this phase, the whole class is evaluated. Here, the aim is to determine whether students have understood the concepts mentioned in the subject, they have learnt the subject and they can apply it to the new situations. For this, students are asked to solve problems related to the cosine law. By examining the students’ answers to the question, teacher can understand how much students have learned. Therefore, teacher can correct the mistakes of students and complete if they haven’t completely learnt. Students’ working papers are examined and evaluated by the teacher.

1. Calculate the angle $\theta$ in the figure below.

![Diagram 1](image1.png)

2. In the adjoining figure, the distance of the reef to two trees is 5 and 8 meter, respectively. Because of the swamp between trees, the distance between them cannot be calculated directly. According to the data given in the figure, calculate the distance between trees.

![Diagram 2](image2.png)